

# QCD's Partner needed for Mass Spectra and Parton Structure Functions

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## Abstract

As in the case of the hydrogen atom, bound-state wave functions are needed to generate hadronic spectra. For this purpose, in 1971, Feynman and his students wrote down a Lorentz-invariant harmonic oscillator equation. This differential equation has one set of solutions satisfying the Lorentz-covariant boundary condition. This covariant set generates Lorentz-invariant mass spectra with their degeneracies. Furthermore, the Lorentz-covariant wave functions allow us to calculate the valence parton distribution by Lorentz-boosting the quark-model wave function from the hadronic rest frame. However, this boosted wave function does not give an accurate parton distribution. The wave function needs QCD corrections to make a contact with the real world. Likewise QCD needs the wave function as a starting point for calculating the parton structure function.

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At the 1965 meeting of the American Physical Society held in Washington, DC, U.S.A. Freeman Dyson stated that quantum electrodynamics can become more effective if combined with other theories [1]. He was right, but he gave a wrong example. He mentioned the calculation of the neutron-proton mass difference by Dahsen and Frautchi as an example. It is still believed that the neutron and proton have the same mass, and the mass difference comes from an electromagnetic perturbation. However, their perturbation calculation uses a non-localized wave function which increases exponentially for large distance [2].

Dyson was still right in saying that QED needs a partner to be most effective. The partner is the localized bound-state wave function. Let us look at the Lamb shift. QED gives to the Coulomb potential a delta function correction at the origin. The S state gets affected by this potential, while the P state is insensitive to this correction at the origin. This results in the shifts between P and S states. The Lamb shift is regarded as one of the triumphs of quantum electrodynamics.

Indeed, in order to calculate the Lamb shift, we need hydrogen wave functions, but quantum electrodynamics cannot produce localized wave functions with probability interpretation. We still have to solve the wave equation with the standing-wave boundary condition to get the Rydberg energy levels and corresponding wave functions.

QED with Feynman diagrams is designed to address scattering problems in the Lorentz-covariant world. The situation is the same in QCD, which is an extension of QED with gluon instead of photons. QCD can make corrections to the existing mass spectra and structure functions, but cannot produce wave functions with proper boundary conditions. Thus, QCD alone cannot produce hadronic mass spectra or parton distributions. It needs a partner.

In 1971, Feynman and his students noted that harmonic oscillator wave functions with their three-dimensional degeneracy can explain the main features of the hadronic spectra [3]. Earlier in 1969 [4], Feynman proposed his parton picture where a fast-moving hadrons appears like a collection of partons with properties quite different from those of the quarks inside a static hadron.

In their 1971 paper [3], Feynman *et al.* wrote down the Lorentz-invariant equation which can be separated into the Klein-Gordon equation for a free hadron, and a harmonic-oscillator equation for the quarks inside the hadron, which determines the hadronic mass. Feynman's equation of 1971 contains both running waves for the hadron and the standing waves for the quarks

Quantum Mechanics	Scattering	Bound States
Galilei	Running Waves	Standing Waves with Boundary Condition
Einstein	Feynman Diagrams	Harmonic Oscillators
	Feynman's One Equation (1971)	

Figure 1: Quantum mechanics in Galilei and Einstein systems. It is possible to construct a Lorentz-covariant model of bound states. Feynman and his students in 1971 wrote down a Lorentz-invariant differential equation which contains both running and standing waves.

inside the hadron, as indicated in Fig. 1.

The oscillator equation takes the form

$$\frac{1}{2} \left[ \left( \frac{\partial}{\partial x_\mu} \right)^2 - x_\mu^2 \right] \psi(x_\mu) = \lambda \psi(x_\mu), \quad (1)$$

where  $x_\mu$  is the four-vector specifying the space-time separation between the quarks. For convenience, we ignore all physical constants such as  $c$ ,  $\hbar$ , as well as the spring constant for the oscillator system.

In the hadronic rest frame, if the time-like excitations are suppressed, this equation produces hadronic mass spectra [3]. If the hadron starts moving along the  $z$  direction, we can separate out the transverse coordinates  $x$  and  $y$ , and write the differential equation of Eq.(1) as

$$\frac{1}{2} \left[ - \left( \frac{\partial}{\partial z} \right)^2 + z^2 + \left( \frac{\partial}{\partial t} \right)^2 - t^2 \right] \psi(z, t) = \lambda \psi(z, t), \quad (2)$$

where  $t$  is the time-separation variable between the quarks. From this equa-

tion, Feynman *et al.* wrote down their solution

$$\psi(z, t) = \exp \left\{ -\frac{1}{2} (z^2 - t^2) \right\}. \quad (3)$$

This form is a Gussied function for the space-like  $z$  coordinate if the time-like variable  $t$  is ignored. It is also invariant under Lorentz boosts along the  $z$  direction. However, due to its non-local time-like distribution, this expression cannot be regarded as a physically meaningful wave function.

On the other hand, this equation also has a solution of the form

$$\psi(z, t) = \exp \left\{ -\frac{1}{2} (z^2 + t^2) \right\}. \quad (4)$$

This solution is Gaussian in both the  $z$  and  $t$  variables. Is it then possible to attach a physical interpretation to this wave function.

First, the time-separation  $t$  exists wherever there is a space separation, according to Einstein. According to quantum mechanics, there is a time-energy uncertainty relation associated with this variable, as shown in Fig. 2.

As Dirac noted in 1927 [5], this time-energy uncertainty does not cause excitations, while Heisenberg's uncertainty generate excitations along the space-like  $z$  axis. However, this space-time asymmetry is quite consistent the internal space-time symmetries dictated by Wigner's little group [6, 7]. According to Wigner, the internal space-time symmetry of massive particles is that of the three-dimensional rotation group without the time variable. We can summarize these in terms of the circle given in Fig. 2.

How about the Lorentz invariance? The form given in Eq.(3) is invariant under Lorentz boosts as  $(z^2 - t^2)$  is. However, the expression  $(z^2 + t^2)$  in Eq.(4) is not invariant. Is this the end of the story? No! Let us boost this form using Dirac's light-cone system [8].

If the hadron moves along the  $z$  direction with the velocity parameter  $\beta$ , the wave function of Eq.(4) becomes

$$\exp \left\{ -\frac{1}{4} \left[ \frac{1-\beta}{1+\beta} (z+t)^2 + \frac{1+\beta}{1-\beta} (z-t)^2 \right] \right\}, \quad (5)$$

This is an elliptic distribution given in Fig. 2, where the circular distribution is modulated by Dirac's light-cone picture of Lorentz boosts. The circle is "squeezed" into the ellipse.

The question is whether we can see the effects of this Lorentz squeeze in the real world. In 1973 [9], in terms of Lorentz-squeezed hadrons, Kim and

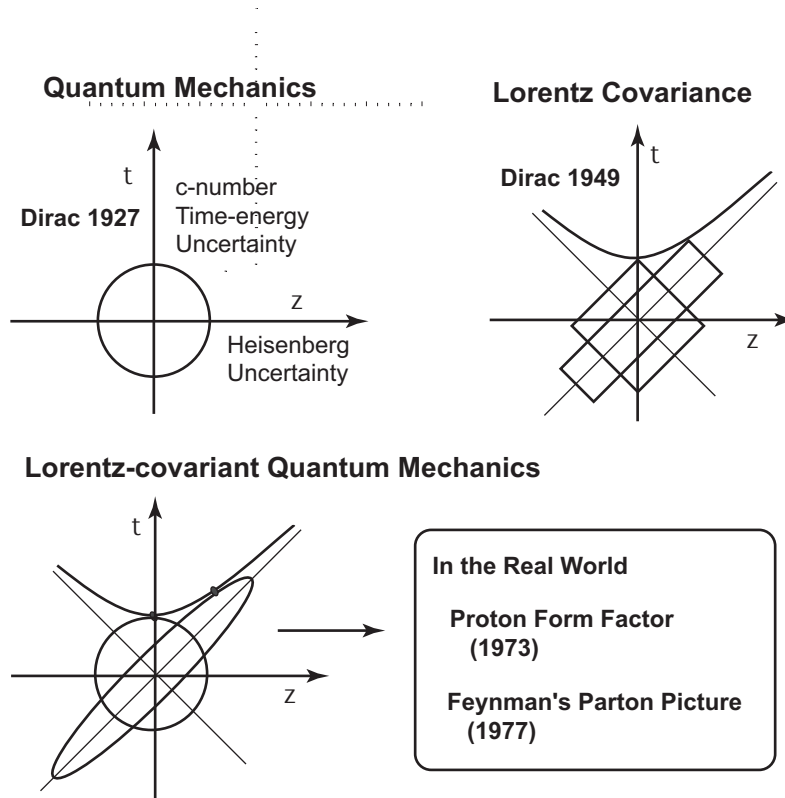


Figure 2: Space-time picture of quantum mechanics. In his 1927, Dirac noted that there is a c-number time-energy uncertainty relation, in addition to Heisenberg's position-momentum uncertainty relations, with quantum excitations. This idea is illustrated in the first figure. In 1949, Dirac produced his light-cone coordinate system as illustrated in the second figure. It is then not difficult to produce the third figure, for a Lorentz-covariant picture of quantum mechanics.

Noz were able to explain the form factor calculation of Fujimura, Kobayashi, and Namiki who derived the dipole cut-off of the proton form factor for large momentum transfers [10].

According to Fig. 2, the quark distribution becomes concentrated along the immediate neighborhood of one of the light cones as the hadronic speed becomes closer to that of light. In 1977 [7, 11], Kim and Noz were able to explain the peculiarities of Feynman's parton picture. Partons have the following peculiar properties.

1. Partons are like free particles, unlike the quarks inside a hadron.
2. The parton distribution function becomes wide-spread as the hadron moves faster. The width of the distribution is proportional to the hadron momentum.
3. The number of partons appears to be infinite.

In the ellipse given in Fig. 2, one of the axis becomes longer while the other becomes shorter. In 2005 [12], Kim and Noz were able to associate these axes as the interaction time between the quarks and the interaction time of one of the quarks with the external signal, respectively. Thus, the external signal is not able to sense other quarks in the hadron. This is what Feynman said in his original papers on the parton model [4].

Kim and Noz indeed explained all the peculiarities of Feynman's parton picture, and proved that the quark model and the parton model are two different manifestations of one Lorentz-covariant entity. However, is it possible to calculate the parton distribution function by boosting the quark wave function from the rest frame? The hadron, when it moves fast, contains both valence partons and gluonic partons. We should therefore obtain the valence parton distribution by boosting the rest-frame wave function.

In 1980 [13], Hwa observed that the external signals do not directly interact with the quarks, but with dressed quarks called valons. Thus, if we remove the valon effect, we should be able to measure the distribution of valence quarks. With this point in mind, Hussar in 1981 compared the parton distribution from the boosted oscillator wave function and the experimentally measured distribution [14]. Hussar's result is given in Fig. 3.

As we can see in this figure, there is a general agreement between the experimental data and the theoretical curve derived from the static quark distribution. Yet, the disagreement is substantial, and this is the gap QCD

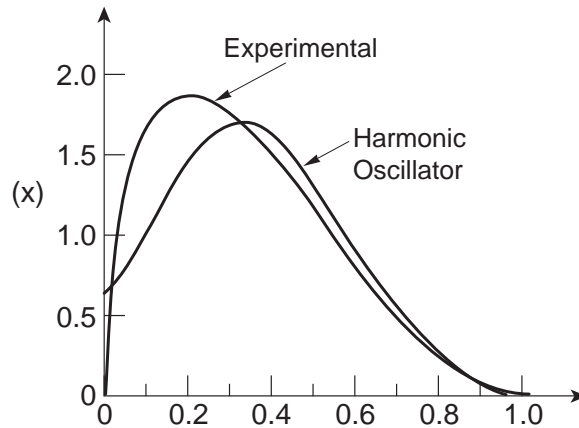


Figure 3: Parton distribution function from Hussar's paper [14]. Although there is a general agreement between theory and experiment, the disagreement is substantial. This difference could be corrected by QCD.

has to feel in. This work is yet to be carried out. The wave function needs QCD to make contacts with the real world. Likewise, QCD needs the wave function as a starting point for calculating the parton distribution. They are need each other. They are the partners.

## References

- [1] F. J. Dyson, *Physics Today*, June 1965, page 21.
- [2] Y. S. Kim, *Phys. Rev.* **142**, 1150 (1966).
- [3] R. P. Feynman, M. Kislinger, and F. Ravndal, *Phys. Rev. D* **3**, 2706 (1971).
- [4] R. P. Feynman, *Phys. Rev. Lett.* **23**, 1415 (1969). See also R. P. Feynman, in *Proceedings of the Third International Conference, Stony Brook, NY, U.S.A.* (ed. C. N. Yang, et al.), pp 237 (Gordon and Breach, New York, 1969).
- [5] P. A. M. Dirac, *Proc. Roy. Soc. (London)* **A114**, 243 (1927).
- [6] E. Wigner, *Ann. Math.* **40**, 149 (1939).

- [7] Y. S. Kim and M. E. Noz, *Theory and Applications of the Poincaré Group* (Reidel, Dordrecht, The Netherlands 1986).
- [8] P. A. M. Dirac, *Rev. Mod. Phys.* **21**, 392 (1945).
- [9] Y. S. Kim and M. E. Noz, *Phys. Rev. D* **8**, 3521 (1973).
- [10] K. Fujimura, T. Kobayashi, and M. Namiki, *Prog. Theor. Phys.* **43**, 73 (1970).
- [11] Y. S. Kim, and M. E. Noz, *Phys. Rev. D* **15**, 335 (1977); Y. S. Kim, *Phys. Rev. Lett.* **63**, 348 (1989).
- [12] Y. S. Kim and M. E. Noz, *J. Opt. B: Quantum and Semiclass. Opt.* **7**, S458 (2005).
- [13] R. Hwa, *Phys. Rev. D* **22**, 759 (1980); R. Hwa and M. S. Zahir, *Phys. Rev. D* **23**, 2539 (1981);
- [14] P. E. Hussar, *Phys. Rev. D* **23**, 2781 (1981).